IASPM JOURNAL
ISSN 2079 - 3871

Considering Golden Section Proportionality in Popular Music: Six Pieces by Jacob Collier

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Abstract

The Golden Section, sometimes referred to as the "golden ratio" or "golden mean", has been referenced in many studies in the arts, serving as a mathematically-based aesthetic criterion for form, structure, proportionality, and balance. In the domain of musical composition, scholars have noted the relationship between the golden section and musical structure; I will briefly illustrate how these studies have focused on proportions and structures in classical music only, which has used a methodology of counting bar numbers in a musical score. I follow this by demonstrating that golden section proportionality may not only be confined to classical music but can also be applied to popular music. I introduce an analytical method that measures the elapsed or clock time of the sounding music rather than examining a musical score. In doing so I use the concept of a temporal "key event point", illustrating this in six works by Jacob Collier.

KEYWORDS: Golden section, Jacob Collier, musical proportion, time-based analysis, music composition

Introduction

It perhaps goes without saying that the field of scholarship in popular music studies has been significantly expanding since from around the time that Wilfrid Mellers authored the *Twilight of the Gods*, his innovative and perhaps idiosyncratic

perspective on the music of the Beatles (Mellers 1973). This growth – accompanied by a proliferation of pop music genres and styles – has incorporated a diverse range of approaches and perspectives in musical understanding and analysis, encompassing cultural theory, ethnography, feminist musicology, semiotics, narrative analysis, as just a few examples. A useful guide through this complex and multi-layered field has been offered by Zagorski-Thomas who notes – in the context of recorded popular music – that "musicology in general, tends to divide into two camps that we might broadly describe as production and reception-based approaches. On one side academics study the technical and creative process that shapes the musical output and on the other they study the way in which audiences (real or idealised) engage with the 'text' to create meaning" (Zagorski-Thomas 2014: 135). It is in the former "camp" that this article sits, aligning with texts such as those by Covach (2005), Middleton (1990), Moore (2003, 2012) Scotto, Smith and Brackett (2019), Spicer and Covach (2010), and Tagg (2009), which are built on formal analytical frameworks originally deriving from studies in classical and contemporary music idioms (discussed at length by Middleton 1990: 103-126). In many ways these texts reflect Middleton's comment on Mellers study of the Beatles in terms of the mode in which they "privilege the parameters of tonality, melodic contour, and, especially, harmony" (Middleton 1990: 113).

However, in this analytical privileging of particular musical parameters, one that receives the least consideration could be said to be the parameter of time; time not in the sense of what one might identify as its "sub-components" – rhythm, duration, tempo, pulse – but time as the space in which the phenomenon of music occurs. Jonathan Kramer describes it thus, "Music unfolds in time. Time unfolds in music" (Kramer 1998: 1) and he quotes Suzanne Langer's well-known phrase that music "makes time audible" (Langer 1953: 110). Kramer makes the point that, "Many scholars agree that time is both the essential component of musical meaning and the vehicle by which music makes its deepest contact with the human spirit. Yet most theorists do not treat time as central to their understanding of music", and suggests that "the majority of theoretical works on musical time deal with rhythm and meter and how they are perceived" (Kramer 1998: 72). Two analytical approaches might be articulated here: firstly, musical time as objective, quantifiable and measurable – generally understood as "clock time", or secondly, music as experienced, understood creating its own temporal reality – sometimes known as ontological time (Kramer 1988: 288). It is the first aspect of time that this paper is concerned with, and from the perspective of the writer/composer, rather than the listener, together with how musical time relates to musical proportion.

Of course, how musical time is articulated or embodied in a piece of music corresponds with the structuring of sections, phrases and other building blocks into formal structures – we see this in classical music forms such as sonata-form, or in popular music such as in verse-chorus form. Form, however, in most studies tends to be treated categorically rather than proportionally; for example, as seen in the area of Covach's extensive codification of rock music structures (Covach 2005), or Everett's analysis of phrase structures and voice leading (Everett 2000). Within this codification there have been scholars who have noted how musical expressivity can be enhanced with the disruption of predictable periodicity or temporal grouping into four-, eight- and sixteen bars: this might occur through unexpected harmonic twists, extended or delayed cadences, a truncated bridge for example

(this can be seen in Holm-Hudson's study of the music of Genesis (Holm-Hudson 2010) and Spicer's essay on "Regatta de Blanc" by The Police (Spicer 2010). Nonetheless, in these academic studies bar-based symmetry and clock-time proportion is not commonly foregrounded, and even here musical proportion is typically discussed descriptively rather than measured systematically as a ratio of total duration.

Proportion and the Golden Section

The Golden Section (sometimes known as the Golden Ratio or Golden Mean and hereafter referred to as the GS), is a geometric/arithmetical concept which can be traced back to the writings of Euclid in the fourth century BC. It describes the division of a single line into two parts, where the ratio of the smaller segment [B] to the larger segment [C] is equivalent to the ratio of the larger segment [A] to the entire line. Mathematically, when considering the whole length as a single unit, the larger segment represents approximately 0.618 of the total length, while the smaller segment accounts for about 0.382, with the two segments together constituting one whole. When expressed as a percentage, the GS occurs at roughly 61.8% of the total length of the line (Figure 1).

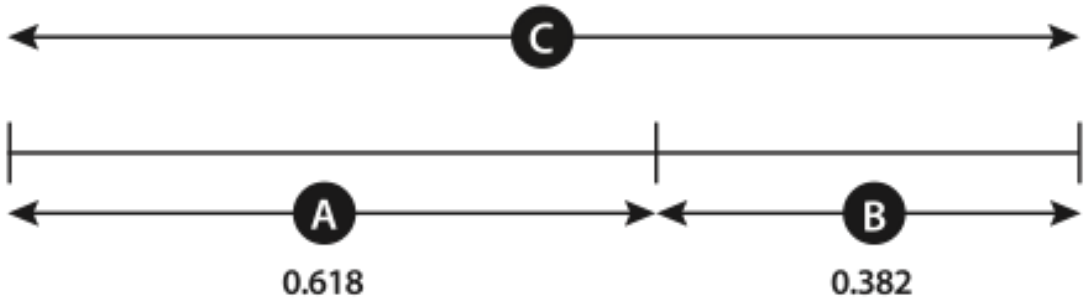


FIGURE 1. Mathematical proportions of the Golden Section

Based on irrational numbers, the GS has been suggested to be a mathematical principle that not only reflects universal laws – such as the structure of DNA, the growth patterns of plants, the shapes of seashells, and the spiral formations of galaxies (for example, Chen et al. 2011; Green 1995; Luttge and Souza 2019; Oldershaw 1982; Zeng and Wang, 2009) – but is also said to be embedded in the products of human creativity. Consequently, it has become associated (although sometimes contested) with concepts of aesthetics: a substantial body of both scholarly and popular literature has explored the role of the GS in the fine and visual arts with many highlighting its evidence in the works of figures such as Leonardo da Vinci (Murtinho 2015), Piet Mondrian (Konečni 2003), Le Corbusier (Arnheim 1966), Hokusai (Evans 2005), with other GS visual arts-related studies (for example, Boseli 1984; Erikson 1986; McWhinnie 1987). An understanding and analysis of the GS in these and other studies has been based on a straightforward methodology which – reflecting the geometric origins of the proportion – involves

the measuring of lengths and proportions within the various visual artistic products. For instance, this might be through calculating the proportions that define the horizon in a landscape, or determining the numerical relationship of different components of a classical group portrait, or within the field of architecture, such as identifying the dimensions of structural elements making up a gothic cathedral or other constructions (Arnheim 1966; Frings 2002). More recently, the visual proportions of the GS have been identified in areas such as computer-aided graphic design (Ying, and Sun Yue 2022), product design (Elam 2001; Gielo-Perczak 2001), packaging design (Raghubir & Greenleaf 2006), as well as car design (Koh 2015) (1).

The Golden Section and Music

In the field of music, and specifically musical composition, there is a not insignificant body of scholarship concerning notions of musical proportion and how it corresponds with the GS. So, prior to considering the golden section in popular music, I aim to offer an overview of these studies, most of which sit in the domain of what we understand as classical music. These musicological studies have their origins in the nineteenth century, most notably in the works of Zeising (1854/2014), who posited that universal laws of beauty – embodied in the golden section – could be identified in both nature and art (Zeising 1855). But, as pointed out by Michelle Phillips (2019), substantial scholarly inquiry into the intersection of music with the golden section only gained real momentum in the latter half of the twentieth century. This was exemplified by extensive studies undertaken by scholars such as Ernest Lendvai and Roy Howat who each authored in-depth monographs of the compositions of Béla Bartók (Lendvai 1966) and Claude Debussy (Howat 1993), although the study of the GS in music actually spans a far wider range of composers over a period of seven centuries. For instance, musicologists have observed the GS in the works of notable composers from the fourteenth and fifteenth centuries, such as Guillaume de Machaut (Powell 1979), John Dunstable (Trowell 1979), and Guillaume Dufay (Sandresky 1981), through Wolfgang Amadeus Mozart (Perry Camp 1968), to more recently with composers such as Erik Satie (Adams 1996), Dmitri Shostakovich (Rofe 2016), Karlheinz Stockhausen (Maconie 2005) and György Ligeti (Luchese 2001).

It is important to note that the method of identifying GS proportionality in the visual arts has been – perhaps unwittingly – directly transposed to all these musical studies with the analytical strategies focussed on the music's "linearity" or, the "horizontal" dimension of music articulated through the temporal and structural evolution of musical lines or sound sequences and manifested as a musical score. This method has been implemented via a methodology of counting barlines and/or individual notes or phrases or sequences within a composer's musical score and then identifying any correlation with the GS. That correlation may occur through musical "markers": throughout much of Western music, the structure, form, or shape of a piece of music is frequently signposted by particular, identifiable and sometimes sudden or immediate, changes that guide listeners through the unfolding musical "territory" or landscape, and it is these that scholars in their studies of musical proportions have employed to identify GS proportions. The musical

markers may be contextualised through the parameters of dynamics, pitch, harmony, tempo, instrumentation or texture and timbre. However, some methodological problems arise; firstly, can simply measuring distances in a piece of visual art such as a painting be simply transposed onto to the study of an art form which relies upon time for its realisation? Howat in his major study of proportions in the music of Debussy, grappled with what analysts should be measuring: bar lines (notation) or the music's temporal durations: "Should temporal proportions in music be measured by clock time or by the music's notated pulse?" (Howat 1983: 15). Secondly, in many of the studies of GS proportionality in classical music, there is the added issue of changes of tempo which may occur within a piece. In his golden section analyses of works by Shostakovich, Rofe underscores this issue of counting barlines stating that, "As there are numerous changes in tempo, bar counting becomes invalid, as the way the music is experienced in time will be conditioned by those changes" (Rofe 2008: 28).

The golden section and popular music: towards an appropriate methodology

As is clear from the above account of the occurrence of the GS in music, all such musicological studies relate to Western classical music; indeed, there is very little to be found in the literature with respect to any music which sits outside this genre and virtually none with respect to popular music. Crucially, all these studies are based on the of counting of barlines in a musical score. However, this ignores the fact that popular music is primarily stored and transmitted in recorded format rather than as a written score; as Tagg points out, "(…) while notation may be a viable starting point for much art music analysis, in that it was the only form of storage of over a millennium, popular music, not least in its Afro-American guises, it is neither conceived nor designed to be stored or distributed as notation (…)" (Tagg 2000: 75). In its recorded form then, popular music is "stored" within the constraints of clock time, within the frame of minutes and seconds and to this author's knowledge, apart from the study by Collins and Dunn (2021) there are no others which aim to understand musical proportion(s) as a measurable distribution of clock time across, for example, a song or instrumental track. Studies such as Covach's *Form in Rock Music* (2005) and Everett's *Expression in Pop-Rock Music* (2000), while exploring common formal designs and phrase structures in rock, only translate periodic organisation and symmetries (using recorded clock timing) at a categorical level, rather than in terms of proportions across a whole piece.

Therefore, I am proposing an alternative methodological approach to exploring golden section proportionality in popular music based on the work of Collins and Dunn (2021) who study GS proportions in measured time across the whole output of the European EDM band, Djihan and Kamien; that is, through a methodology of measuring chronological time – or clock or elapsed time – rather than through the analysis of a musical score.

Jacob Collier

Stumbling one day across the quite remarkable arrangement of Henry Mancini's "Moon River" by Jacob Collier – an artist of whom I previously knew very little – led me over a period of time to explore his other musical output. Collier is an English singer-songwriter who is also a multi-instrumentalist, performer, producer, and prolific contributor to online discussions of his harmonic vocabulary (such as microtonality, shifts between equal or just temperament, chord clusters, unusual modulation shifts) as well as regularly delivering workshops at MIT and elsewhere. He is also the composer, performer and producer of some fifty tracks on a four-volume album, *Djesse Vol. 1–4* (2018-2024). On these albums, which cross popular music, jazz, rock, choral, digital, and gospel music among other idioms, Collier performs with an extremely wide range of artists as well as with choirs and orchestras, collaborating also with audio-media technologists to help realise his musical intentions. Each album has secured him a Grammy Award, on top of two Grammys for songs on his debut 2016 album *In My Room*, an album composed, performed and produced by Collier in the back room of his home in London. Most recently the seventh Grammy was awarded for his arrangement of Simon and Garfunkel's "Bridge Over Troubled Water" (2024).

Delving into Collier's sound world it became increasingly clear that some of his approaches to traditional and non-traditional musical forms and harmonic language corresponded with the proportional attributes of the Golden Section. To my knowledge there are no published analyses of his music: here I aim to introduce a case study analysis of three of his original compositions and three of his arrangements of mainstream songs which I suggest align with golden section temporal proportionality.

Six pieces by Jacob Collier

From the range of around sixty-four tracks by Collier, which cross five albums, these six pieces represent a very broad range of styles, from a straightforward popular song ("Little Blue") to an *a capella* choral work ("Home Is"), a jazz-oriented work ("Don't You Know") to three extended arrangements of popular classics ("All Night Long", "Moon River" and "Bridge Over Troubled Water").

The analysis is supported with extracts from Jacob Collier's own discussions of his songs on YouTube. In the analyses following, the total duration of a track is measured in seconds, and then multiplied by the golden section constant (0.618) to determine where the "positive" (2) GS point would lie. In other words, for a hypothetical five-minute piece, calculating the GS point would be through converting the five minutes into seconds (=300) then using the GS multiplier of 0.618; thus, the CGS would occur at 185.4 seconds into the piece. Since Collier's mode of music creation is primarily through digital audio workstation (DAW) software tools such as Logic, which rely upon graphical rather than conventional music notation, the scored examples of his music in this paper are transcriptions with corresponding bar numberings used for illustrative/referencing purposes only, avoiding the inclusion of a screenshot of the DAW's arrange window (Collier's

preferred compositional environment) from a composition such as "Moon River" which has over 100 MIDI and audio tracks.

Little Blue

"Little Blue" is included on Collier's album *Djesse Vol. 4*, released on 29 February 2024, featuring the vocals of Brandi Carlile, and the Aeolians choir with all instruments (acoustic guitar, bass, balalaika, keyboards, percussion) played by Collier. This relatively formally straightforward song, in comparison with the harmonic complexity of some of Collier's other works, remains in a consistent overall key and a consistent tempo throughout. In comparison with the dramatic key event point in "Moon River" which I describe later, with its sudden shifts of tonality, and a transition to just intonation from equal temperament, the key event point in this particular song is much more subdued.

The song is 4′25″ long and so the calculated or mathematical golden section (hereafter referred to as CGS) lies at 2′43″. What happens very close to this point is that the acoustic guitar, balalaika, electric bass, pedal bass, choir, and shaker – which have been playing consistently up to here – all drop out and Collier quietly introduces a sample of his "audience choir" at 2′39″. This is a significant moment for Collier since the audience choir is now an established part of virtually every Collier live concert, where he leads the audience in singing single sustained tones in simple two- or three-part harmonies. Drawn from live international performances, Collier introduces a combination of audience recordings for this particular piece from three concerts: Vienna, 4 November 2022, Sydney, 8 December 2022, and Brisbane, 10 December 2022. The significance of this moment in "Little Blue" is emphasised by Collier in his YouTube commentary: "I combined these three audience choirs, chopped them up, put them into Logic…I added some reverb, and these three choirs become the underpinning voices behind my new song Little Blue" (Collier 2023). These audience voices, which enter at 2′39″, continue swelling in and out until the end of the song; in terms of GS proportions as measured in clock time, this key expressive moment where Collier "underpins" his track, can be seen to constitute just a brief four second deviation from the CGS point or minus 1.88% deviation across the whole song (Figure 2).

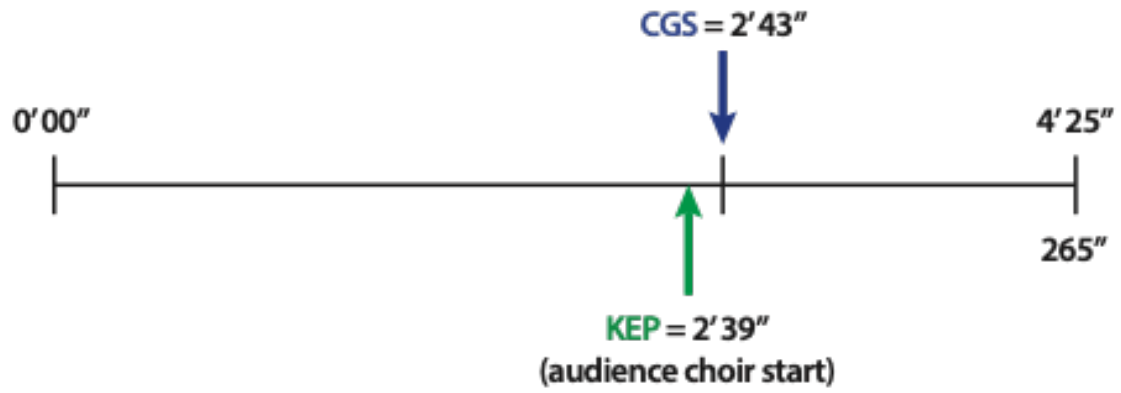
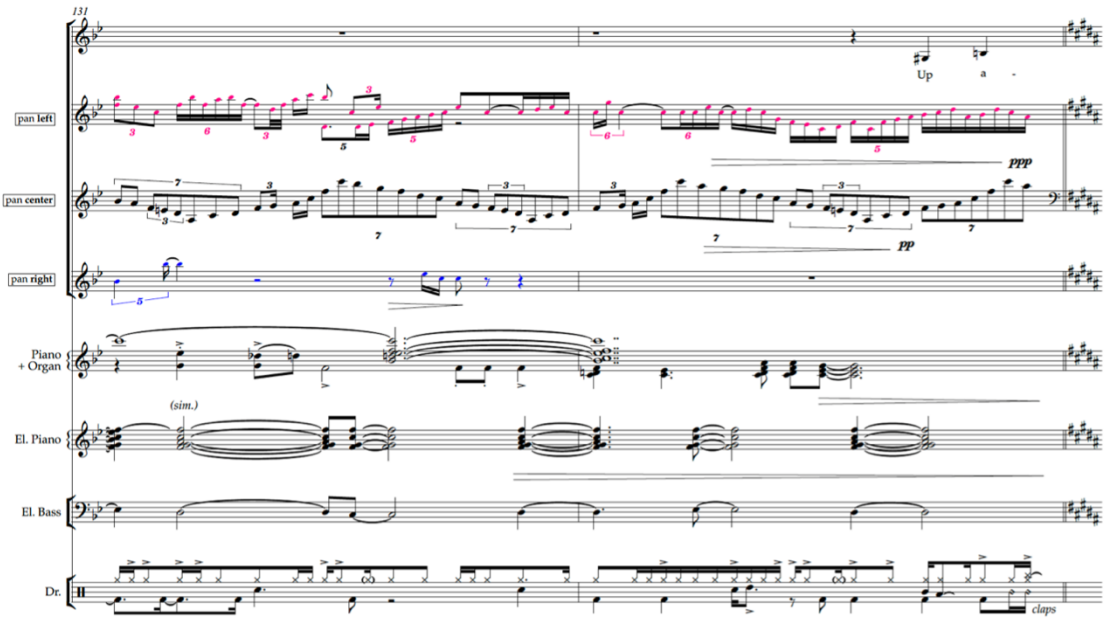


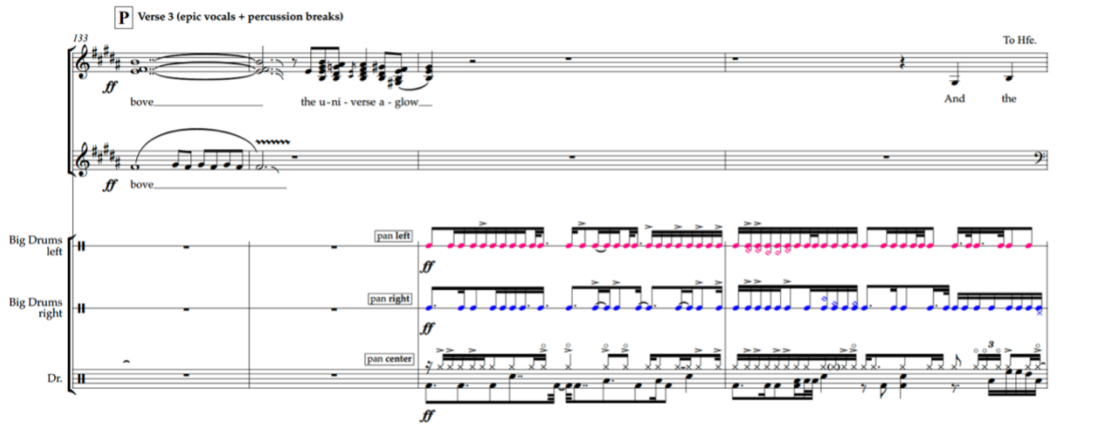
FIGURE 2. The key event point of "Little Blue" at the start of the audience choir.

Don't You Know

The methodology of identifying GS points in clock time rather than through the conventional mode of counting bars, can be realised in quite a different genre: "Don't You Know" with music and lyrics by Collier, is a track on his 2016 album *In My Room*, with Jacob Collier playing all the instruments; this also features on a live version on the *Family Dinner – Volume 2* (2016) by Snarky Puppy, a contemporary jazz, Latin jazz, funk-based Snarky Puppy band with Collier guesting on keyboards. In both versions the song is set in B flat major with a fairly conventional structure of intro, verse/chorus (x 3), instrumental and outro, and in the live version outlined below, lasts 10'43''.

Just over six minutes into the piece following the second chorus, an extended piano-dominated instrumental interlude led by Collier drops onto a twenty-second-long quasi-dominant pedal on pentatonic F, until at 6'32" there is an unexpected shift not to the expected B flat major opening for the third chorus but to the major pentatonic on E. This surprising direct modulation, with its opening three note "sticks and stones" sung in the first verse as g, b flat, f (now having become g sharp, b natural, f sharp) (Figures 3 & 4), constitutes a jump of an augmented fourth and represents the key event point in the whole piece lying just five seconds away from the mathematical golden section point of 6'37''. Quite remarkably – since this is the only live studio track of the six pieces represented in this study – the overall temporal "distance" between the calculated (for example, mathematical GS point) and the key event point is a mere 0.46% (Figure 5).





FIGURES 3 & 4. The shift from B flat major to E major (transcription by Lukas Lessing, 2021).

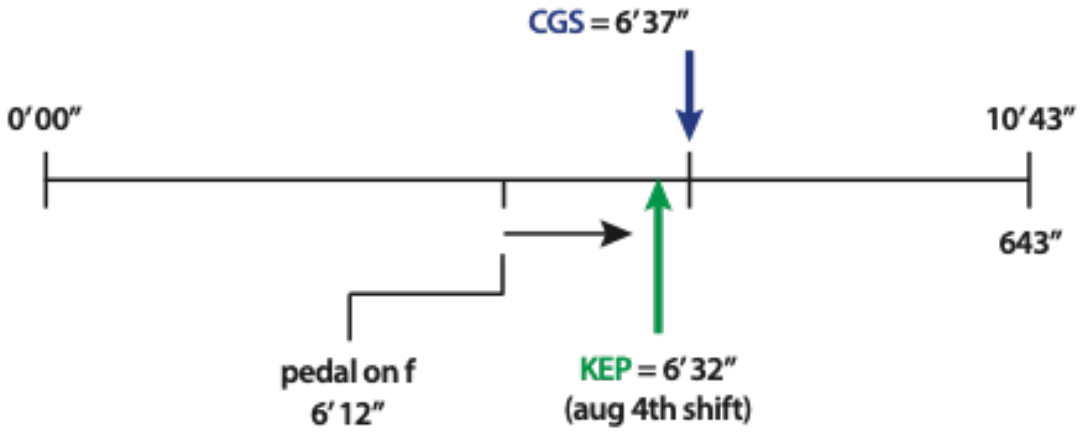


FIGURE 5. The Golden section entry in "Don't You Know" with the shift from the key B flat to E.

Home is

"Home Is", on the *Djesse Vol. 1* album (2018), represents a markedly different style of composition for Jacob Collier, in that it is fully scored in conventional notation, only mirrored by his piece for unaccompanied choir "World O World" on *Djesse Vol. 4*. Performed on the first of Collier's *Djesse* albums by Voces 8, its forty-three bars last nearly six minutes at crotchet = forty, and is unusual in the respect that there are just two words vocalised within the entire piece, and with a very minimal addition from Collier on piano and percussion. In many respects, the piece evokes the aesthetic and stylistic qualities of choral works such as William Byrd or Thomas Tallis, or more recently, Eric Whitacre. Although scored in conventional music

notation, it is very likely that the piece would have been composed, notated and edited within the DAW environment even though ultimately deploying a live choir.

Following a gradually unfolding opening, with short musical phrases punctuated by long silences, and the choir singing wordlessly or just intoning the single word "Home", the piece builds up slowly through a sharp key, C sharp major. Then, following a long seven second silence after the humming of the lowest bass note in the whole piece, C sharp, has died down, Collier suddenly pivots the voices away from C sharp major into a flat key, A flat major. He achieves this by using G sharp, the fifth of the previous key, as an enharmonic link with the tonic of the new key. At the same time, the choir enunciates for the first time the two words "Home Is". This immediate shift from a sharp to flat key is assisted and smoothed by the use of subtle microtonal directions to the singers, which can be seen at bars 25-28 with corresponding directions to lower the pitch of the G sharp in the E major chord in the bass voices by fourteen cents, and to raise the G sharp of the following C sharp major chord by two cents. With the differential between just intonation and equal temperament of a major third being 13.68 cents (Withington 2020), this direction to the vocalists reflects Collier's liking for just intonation as a highlighting device. There is also an additional indicator that this marks the end of a particular expressive landscape; as the C sharp major hummed chord fades, Collier adds a single *sotto voce* timpani hit. Thus, the unexpected enharmonic modulation to A flat major has the effect of what Martin describes as, "occupying a new space within the emotional landscape that feels different somehow" (Martin 2020) (Figure 6).

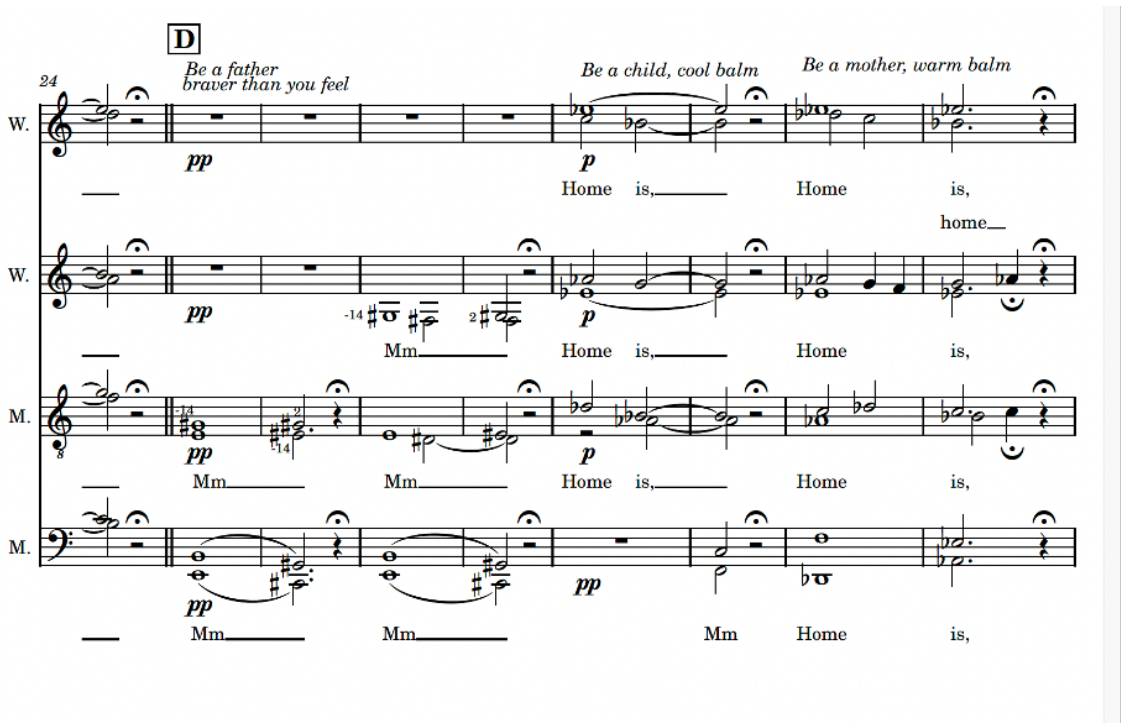


FIGURE 6. Extract from "Home Is" with the enharmonic transition to A flat major at 3'31". Transcription by Barnaby Martin (2021).

This significant and highly expressive point in the whole piece, is constructed through Collier's combinatorial use of silence, a sudden shift from sharp to flat tonality, the first vocal enunciation of "Home is", and the microtonal adjustment to just temperament for the voices and occurs in clock time just three seconds prior to the calculated golden section point of 3'33'' (5'45" = 345" x 0.618 = 213" = 3'33"). Employing a temporal counting of minutes and seconds leads to an overall deviation between the musical and mathematical GS points of just 0.86% deviation (Figure 7), not dissimilar to "Don't You Know" in its close proximity.

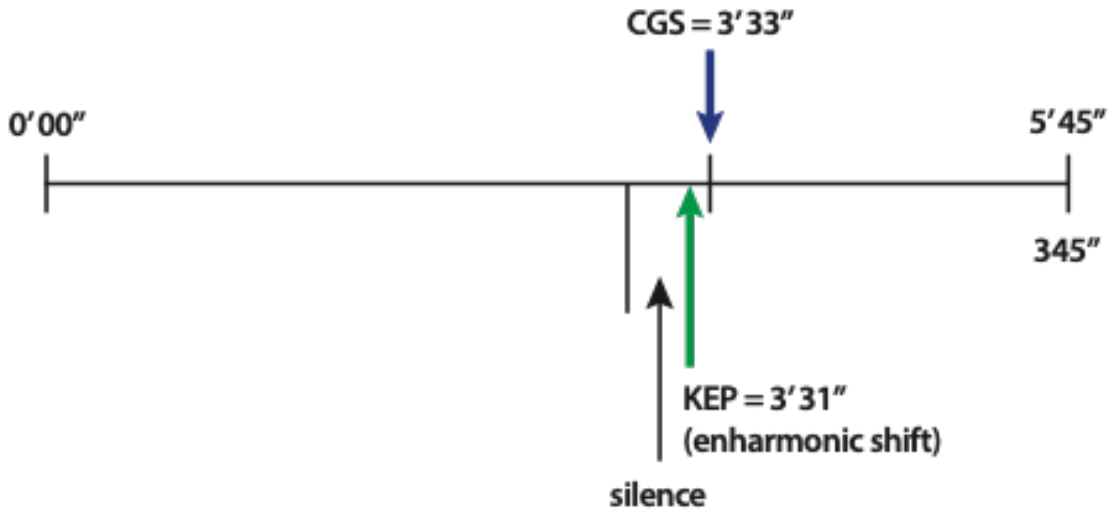


FIGURE 7. The key event point in "Home Is" at the enharmonic shift from sharp to flat.

All Night Long

Found alongside "Home Is" on the *Djesse Vol. 1* album (2018), this arrangement of Lionel Ritchie's well-known 1983 hit incorporates, as part of Collier's DAW-based music production, the Metropole Orkest and Take 6, the *a cappella* gospel-oriented group and the choral group the Aeolians. While Ritchie's original recording is just under four minutes in length, Collier expands this to over seven minutes in a complex instrumental and vocal arrangement, together with improvised jazz-based breaks by Collier counterpointing on keyboard and bass guitar against a brass section, Take 6 and a large vocal section and the orchestra – the album credits indicate approximately 150 musicians involved in the production. The complex Latin jazz arrangement rotates around a range of key centres and extended jazz-oriented harmonic sequences, but at 3'53" the percussion comes to a halt, followed by a fifteen second massive dominant pedal crescendo with an accumulation of instrumental build-up in the orchestra and choir with its climax on the Aeolians' choral exclamation accompanied by brass riffs; if we regard this intense culmination of the build-up as representing a key event point, then clock time measuring places it at 4'30" while mathematically the GS is at 4'31" – in other words, a very small deviation across the piece of -0.22% (Figure 8). After this point

the percussion returns on bongos at 4′47″ with an orchestral mashup following until the fadeout. In his YouTube "Logic Session Breakdown", Collier refers to this GS point as that place where, "energy [is] growing and growing up to this moment which is such a fun moment" (Collier 2019b).

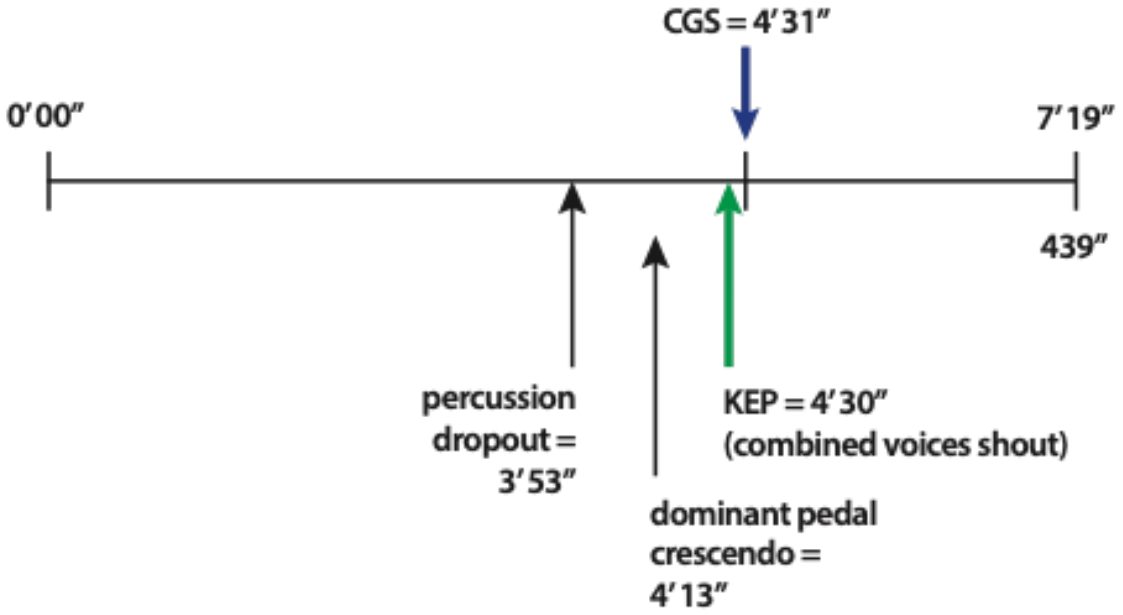


FIGURE 8. The key event point of "All Night Long" at the extended buildup.

Bridge Over Troubled Water

In one of his YouTube "Logic Breakdown" sessions, Collier (2024) explains that his arrangement of this song first began in 2021 when he came across an online video of Yebba singing this Simon and Garfunkel classic; accordingly, for his aspiration of wanting to "do something on a bigger scale" for his *Djesse Vol. 4* release, this version of the song was realised. The scale of the arrangement can be confirmed simply through the sheer number of vocal tracks used on the DAW, that is, some 316. On the YouTube breakdown Collier describes the broad ground plan that he had for the structure of the piece which was heavily influenced by his choice of the three vocalists he chose prior to composition. He sets the three verses of the song in different keys for each of the soloists:

Verse 1 sung by Yabba – A flat major
Verse 2 sung by John Legend – E flat major
Verse 3 sung by Tori Kelly – B major

In the preparation for the final verse, which now closely corresponds to the CGS, Collier states that he had "this big ambition to go to all these different keys [with] quite a specific progression". In effect this is realised as a dramatic shift away from E flat major which closes the second verse sung by John Legend, onto B major, an augmented fifth away. Condensed into just a few bars, the final chords leading out of verse two end on C natural thirteen sharp eleven (C13♯11), where the B flat pivots

enharmonically as an A sharp onto the seventh of the B major added ninth on a pause. Collier describes the emotional impact of this point in the composition: "that chord [B major] means that we're about to be somewhere satisfying…and sure enough, B major …[and] there's a moment of silence here" and that, "the D sharp on the B major chord is dark and flat and justly tuned". Tori Kelly's voice then enters onto a hugely pared down minimalist arrangement, on the words "Sail on, silver girl. Sail on by. Your time has come to shine" (Figure 9). And in order to further emphasise the distinctiveness of this key point in the whole composition Collier goes on to state that, "I made sure that this first phrase with Tori was totally *senza vib* – flat as a pancake" (Collier 2024). If we consider this as a key event point – the combination of a transition to the final verse, a shift to a distant key and a radical change in vocalisation prefaced by a silence – then the entry of the words, "Sail on silver bird" temporally sits just 1.94% away from the calculated golden section point for the piece (Figure 10).

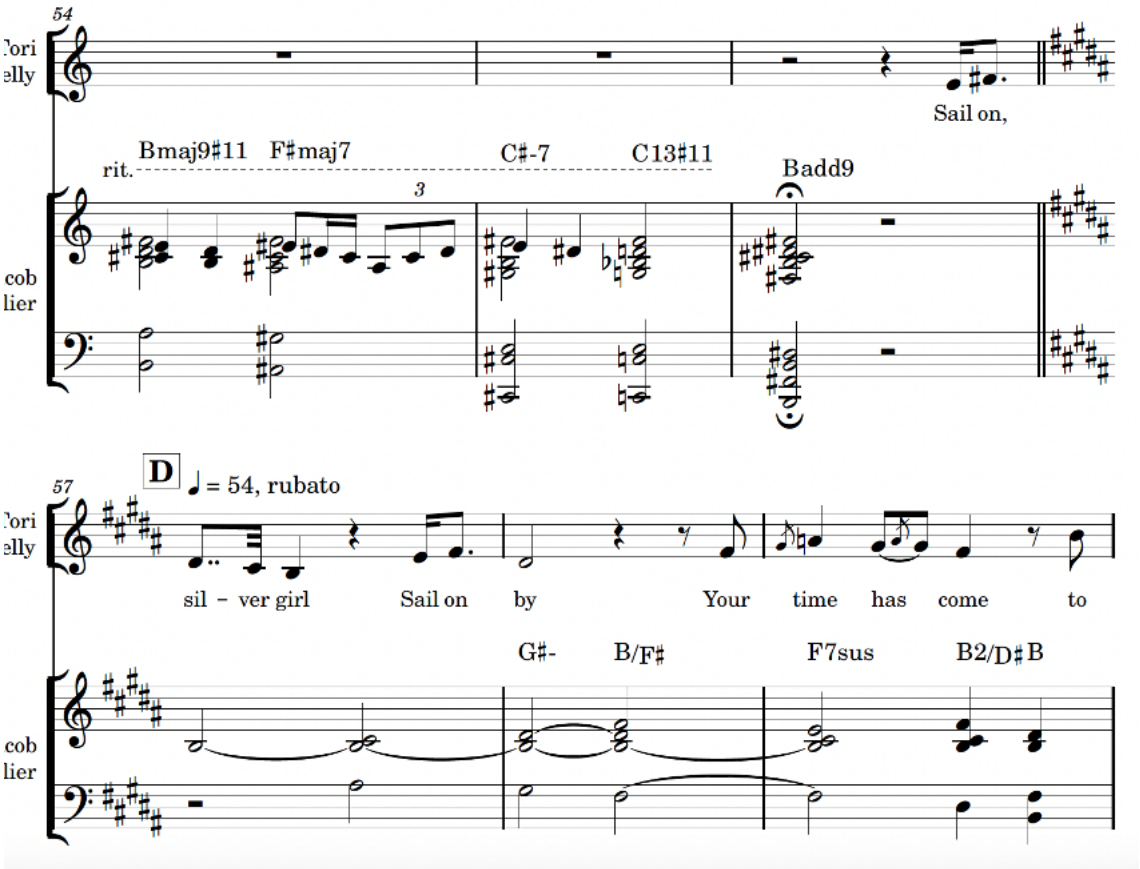


FIGURE 9. Extract from the close of verse 2 of "Bridge Over Troubled Water" transitioning into B major. Since this piece is set across a steady pulse at approx. crotchet = 56 (rubato), then counting bars of the transcription also leads to an identical key event point at bars 56/57 (total bars = 88 x 0.618). Transcription by Felix Broman.

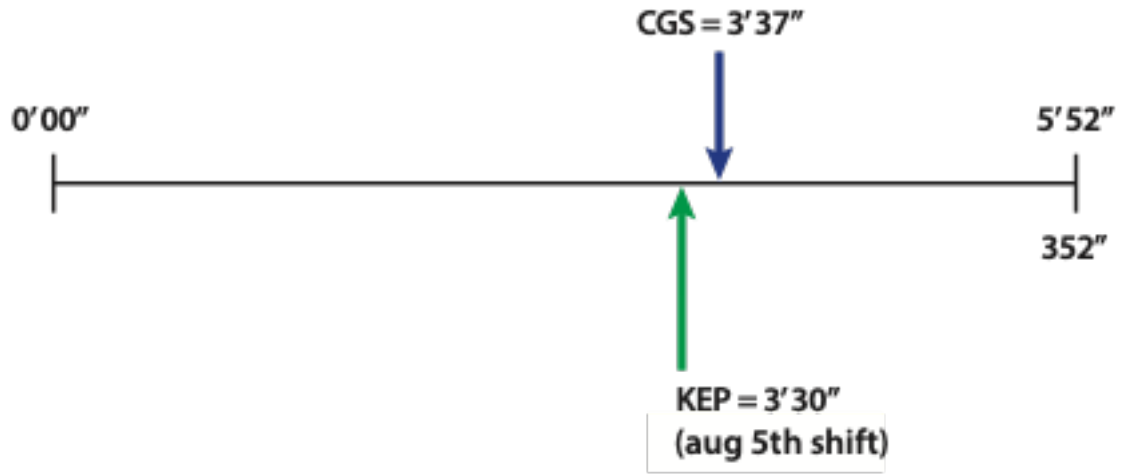


FIGURE 10. The key event point of "Bridge Over Troubled Water" at the beginning of the third verse

Moon River

"Moon River", from *Djesse Vol. 2* (2019) is a radical reformulation and reworking of the classic song by Henry Mancini, with lyrics by Johnny Mercer, originally performed by Audrey Hepburn in the 1961 film *Breakfast at Tiffany's* and popularised by Andy Williams. The original song for the film, simply scored for voice, acoustic guitar and string orchestra is a little over two minutes long; Collier's version on the *Djesse Vol. 2* album from 2019 is four times that, and the pared-down scoring by Mancini is now expanded to up to a 100 layered voices. For this remarkable version of the original song, Collier used a DAW to incorporate both live and MIDI sounds. Structurally speaking it consists of an extended lead in with arhythmic sampled vocals intoning the word "moon", followed by three verses of the song each in a different key. Martin describes the piece as "pushing every aspect of his work to its limit. It contains, microtonality, alternative tuning, eight different keys or pitch centres as well as some of the most complex and dense harmonic language he has ever used" (Martin 2019).

In his YouTube "Logic Session Breakdown" (Collier 2019a), Collier "deconstructs" his composition, outlining his compositional processes and thinking. In this narrative, he describes the critical importance of the last verse in the overall composition, and I suggest, through outlining the compositional procedures he uses, that this is the location of the key event point. Up to this point within the whole arrangement, the extended introduction had begun in B flat major, with the first verse in D flat major, and the second verse in D major. Then, in the transition to the third verse, there occurs a passage which is what Martin describes as, "possibly one of the most developed and complex in all of Jacob Collier's music". In essence, Collier travels from D major through the flat keys of B flat and E flat, then quickly through F sharp major, B minor, and A major finally arriving in the region of B flat. Here we see the shift from A major to B flat major enabled by using a tuning not of A4 = 440Hz but rather A4 = 432Hz. Martin points out that this is "a completely unprepared shift" and since there is "only a small distance

between A in 440 and B flat in 432, the change in pitch is completely disguised". After this densely layered quasi-dominant seventh chord of B flat seven the music then "resolves" onto E flat in this new reference pitch which is, given the microtonal adjustments, essentially D half sharp. In his Logic Breakdown analysis Collier comments that the piece is "entering into the key of D half sharp (…) it's such an exciting place to be" (Collier 2019a). This harmonic transitioning allied to the microtonal shift, is to him a way of "announcing this huge moment" which to him is "really cathartic". On entering this last verse, he says that he has acted to "pull out all the stops" which he achieves primarily through recording ten vocal bass parts as a massive sonic underpinning to the introduction of the final key (bar ninety in the transcription below). He likens this moment in the overall structure as "a totalistic pillar, a totem pole (…) a sense of direction"; this is prefaced by a dominant cluster chord, where half-sharp notes mix with standard tuning (Figure 11). We are therefore now at a significant expressive moment – a key event point – occurring at 5'20" (or bar ninety in the transcription) which represents a temporal deviation from the calculated golden section of fourteen seconds within the eight-minute piece (for example, 496" x 0.618 = 306" or 5'06"), a small 2.82% overall deviation (Figure 12).

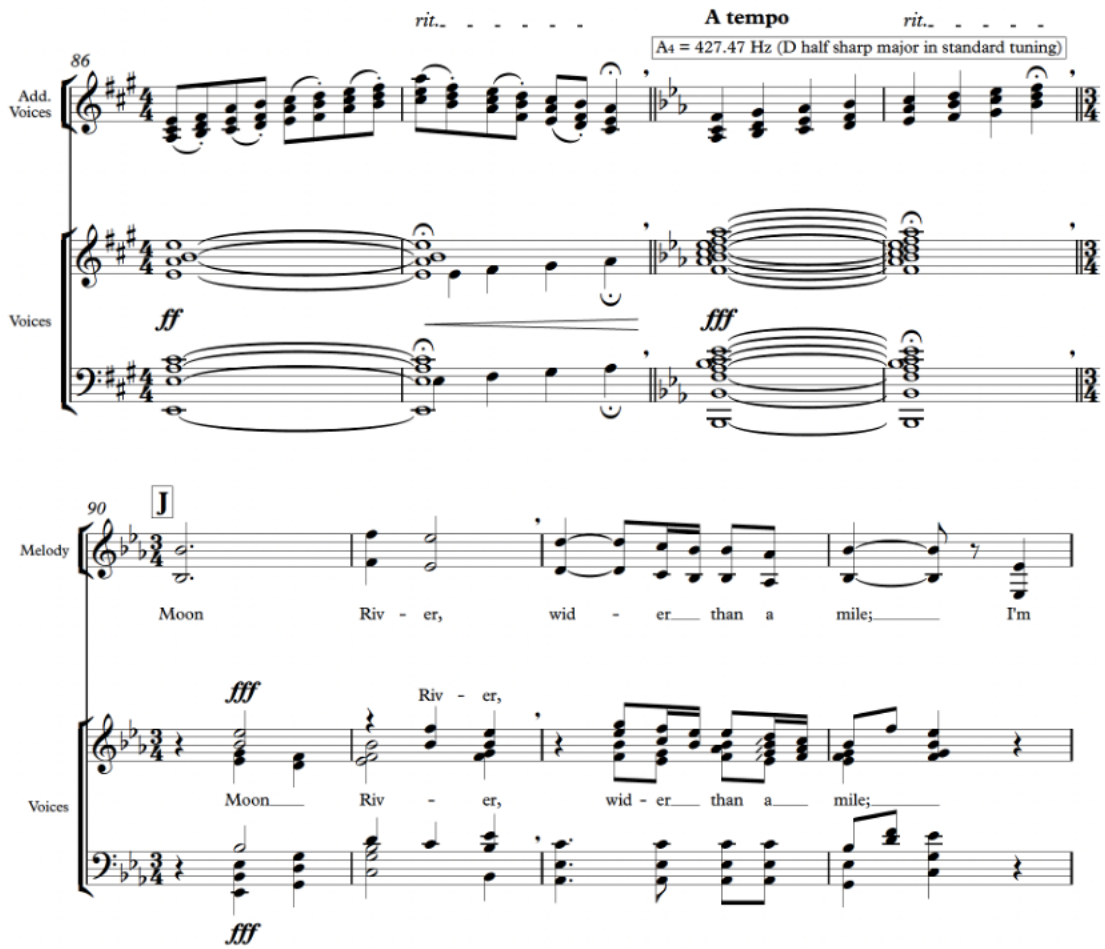


FIGURE 11. The final verse which has landed on E flat major. Transcription courtesy of June Lee (2019). His transcription indicates E flat major as D half sharp major.

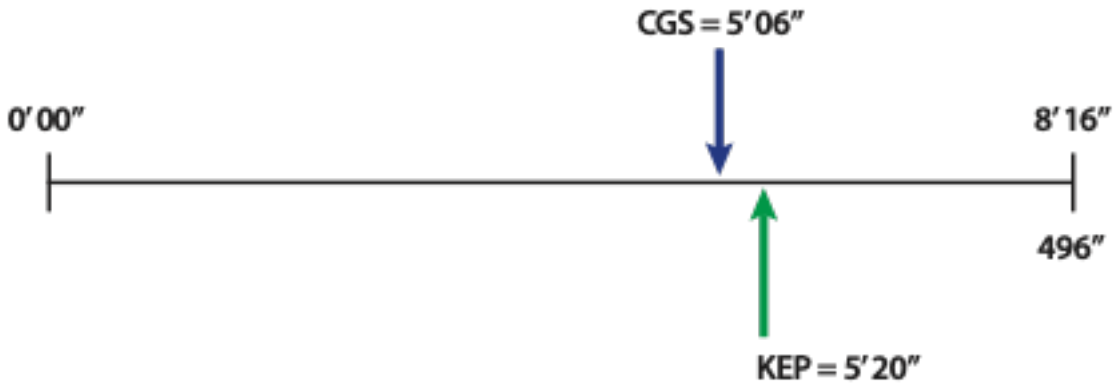


FIGURE 12. The key event point of "Moon River" at the point of the entry into D half sharp major.

The issue of temporal deviation

When considering the significance of a key event point within a musical composition, it is crucial to consider what are acceptable temporal deviations from the mathematically calculated GS point. This is not so straightforward, since as Žuvela points out, "The irrationality of the Golden Ratio constant causes further problems with its application in a discrete medium: it can never be embodied with complete accuracy. Theorists and analysts are therefore often in dispute over the threshold of tolerance of the deviation from the ideal proportion and the determination of a measurement unit sensitive enough but not too small to indicate the significance of the proportion" (Žuvela 2011: 277). Bearing this caution in mind, and looking to earlier GS studies in classical music, we can see that, for example, in his analysis of Puccini's Act 1 love duet in La Bohème, Atlas suggests a deviation up to 2.6% might be acceptable (Atlas 2003: 277), and in his exploration of the GS in the music of Shostakovich, Rofe offers a deviation of between 1.5 to 2.5% (Rofe 2008: 32) as being appropriate.

With reference to the GS in popular music, this deviation figure is mirrored in the non-score-based methodology study of Collins and Dunn (2021) where the researchers noted that the majority of forty-five tracks from albums by Dzihan and Kamien were "well below" (2021: 122) the researchers' set deviation threshold of 3.5%: in the six works by Collier outlined above, the overall deviation of the key event point from the calculated golden section point lies between 0.22% and 2.82% (Table 1). Adams states that GS proportions might be "perilously close" to the common division of one third/two thirds in much music (Adams 1996: 243). However, by way of example, in "Little Blue" there is a 1.5% deviation, which corresponds to just four seconds between the CGS and the KEP. Had the "rule of thirds" applied, then this would conversely translate to any significant musical event (for example the middle eight) lying some twenty-four seconds away from the CGS; in the case of "Don't You Know", this point would sit twenty-seven seconds away rather than the observed GS point of three seconds; and in the case of "All Night Long", two thirds of the way into the song would translate into an eighteen

second deviation rather than one second from the calculated Golden Section. Hence, it is difficult to concur with Adams' concern that, "Slightly overshooting the first [third] or undershooting the second can easily bring the composer into the purview of GS" (Adams 1996: 243-244). It may be useful to recall that Adams' calculations were predicated on a methodology of counting bars rather than clock time. In this context, Winterson points out that the verse/chorus, one-third/two-thirds convention in popular music "is the approximate point where the middle eight, a contrasting section, is most frequently to be found" and in suggesting that this is commensurate with GS proportions says that, "A clear example of GS in pop music can be found in Lady Gaga's 2016 song 'Perfect Illusion' where there is a dramatic key change at exactly this point; the song is 179 seconds long and the key change happens at 111 seconds (179 x 0.618 = 110.622)" (Winterson 2024: 161). However, the two-thirds point would actually lie at 179 x 0.66 = 118, or some seven seconds away from the GS point.

To illustrate this point, the following table indicates the deviation in seconds of the key event point from the calculated GS in each of Collier's six pieces; the number in brackets is the alternative deviation point predicated on the two-thirds/one-third basis:

TABLE 1. Temporal deviation in seconds in ascending order, all data a-d in seconds.
a = length of song
b = calculated golden section (CGS) (a x 0.618)
c = key event point (KEP)
d = differential between b and c in seconds
e = deviation as a percentage of total duration from b [d/a x100]

	a	b	c	d	e
All Night Long	439	271	270 (289)	1 (18)	0.22
Don't you know	643	397	394 (424)	3 (27)	0.46
Home is	345	213	210 (227)	3 (14)	0.86
Little Blue	265	163	159 (175)	4 (12)	1.5
Bridge over troubled water	352	217	210 (232)	7 (15)	1.94

Moon River	496	306	320 (327)	14 (21)	2.82

Finally, this exploration of GS proportionality in the music of Jacob Collier has been supported by Collier's own narrative in his online "Logic Breakdown Sessions" where he sometimes spends up to an hour describing his compositional processes at the DAW. These have been invaluable in illuminating the analysis presented here, and I have presented the relevant commentary at Table 2. It can be seen that Collier has a liking for marking the key event point with unexpected harmonic shifts to unrelated keys and new instrumental/vocal entries acting as highlighting and expressive devices.

TABLE 2. Summary of musical markers for GS event points.

	Key event point description	Collier commentary
Little Blue	Changed arrangement Entry of audience choir	"these three choirs become the underpinning voices"
Don't You Know	Shift to distant key (augmented fourth to E major pentatonic) after pedal on F.	n/a
Home Is	Choral enunciation of words together with an enharmonic shift from sharp to flat key.	n/a
All Night Long	Extended dominant pedal crescendo	"energy growing and growing up to this moment which is such a fun moment"
Bridge Over Troubled Water	Shift to distant key (augmented fifth to B major) at beginning of third verse, with introduction of female vocals	"that chord means that we're about to be somewhere satisfying (…) and sure enough, B major (...) [and] there's a moment of silence here"
Moon River	Shift to distant key (augmented fourth to D half sharp/ E flat major: beginning of third verse	"announcing this huge moment" (…) [which is] really cathartic (…) [and where I] "pull out all the stops"

Conclusion

From the analysis of these six diverse pieces by Jacob Collier some considerations emerge. Firstly, as I have indicated earlier, it is necessary to question whether the established methodology observed in many studies – in the realm of classical music – of counting bars in a musical score to determine golden section proportionality is

an appropriate methodology to apply to popular music, which is rarely contingent upon the initial production of a musical score. While this counting methodology has been generally accepted by scholars as being isomorphically congruent with the methodology of measuring distances and lengths to assess the location of the golden section in a visual artefact, it ignores the notion that in popular music our perception and understanding of recorded music exists temporally rather than via a musical score. This latter is described by Tagg as "notational centricity" where "the score is treated as a reification of the composition" (Tagg 1987: 5). Certainly, we can see that through the non-score-based methodology adopted here, there is a real opportunity to explore GS proportionality in temporal terms.

The question may then arise as to whether the listener can actually apprehend temporal GS proportions in a musical work; the comprehension of proportions within an unfolding *gestalt* of a piece of music, in comparison with viewing a painting, an architectural form or other spatial artefact, may be more problematic, since, as Reybrouck has suggested, "music, as a temporal art, is essentially discursive. In contrast with a geometrical figure, that is described as a whole when looking at it, a musical figure needs a successive presentation" (Reybrouck 1997: 64). As Atlas drily suggests, this is "a far more complicated affair than that of spatial proportions, which are nice enough to stand still" (Atlas 2003: 282). While there are empirical studies regarding visual perception of the GS (for example, McManus and Weatherby 1997; McWhinnie 1987; and Benjafield's useful 2010 metastudy) there are no comparable studies, to this author's knowledge, in the field of music perception. This issue may be an analytical *cul-de-sac* however; the structural perception of a piece of music can only be an after-the-event phenomenon since, as Boykan points out: "you can easily compare lengths in the visual world, but it is rather a stretch to ask us to keep track of a ratio in a situation of gradual unfolding" (Boykan 2004: 25-26) a point echoed by Phillips (2019). Nonetheless, further exploration of listeners' temporal apprehension of GS proportions in a piece of music may yet yield interesting insights.

As any discussion of how any golden section in popular music may be perceived is beyond the scope of this paper, this too applies to the matter of compositional intentionality: are writers consciously aware of the GS ratio while putting together a piece of music, a song, or instrumental track for instance? Some earlier classical-based studies have suggested that there has been some conscious intentionality by different composers; for example Howat (1993: 164-168) discusses the possibility that Debussy may have been influenced by musical proportionality through his circle of friends in Paris, as well as the contemporary Symbolist movement in the visual arts, and there is some evidence from Bartók's work on Turkish folk songs that may offer evidence of deliberate mathematical/proportional construction (Winterson 2025: 169-170). Adams (1996) explores a range of possibilities that Satie may have learned of the GS from a range of non-musical sources, but concludes that there is no evidence to prove that Satie actually knew of the GS. More recently, Collins and Dunn (2021) in their case-study asked their composers/producers whether they had deliberately aligned some forty-five of their musical tracks with the GS, but this too yielded an unclear outcome (2021: 124). And there is nothing in Collier's many YouTube presentations which would indicate that he had deliberately intended to associate his creative output with GS

proportions. Composer intentionality and the GS could well be another fruitful area for further exploration.

Given the fact that Jacob Collier almost exclusively writes his music using a DAW, the further question may arise as to whether or not this compositional affordance introduces a certain degree of sensory interchange between the visual and the musical/aural domains. In other words, we might consider whether these screen-based tools might, as Collins and Dunn suggest, "naturally engage a visual, holistic appreciation of possible golden section proportions upon the computer screen which might then in turn be translated into temporal music and sound events" (2011: 123). Macchiusi captures such possibilities in his account of the visualisation of sound describing the affordance of the DAW in comparison to a conventional musical score: "While being spatially vague in regard to time, a musical score is not reductive enough to capture the broad sweeps of the arrange window's (3) synoptic representation" (Macchiusi 2017: 129). It may well be that this visual, synoptic overview of an emerging composition affords the artist working with digital tools that enhanced permeability between the visual and the aural where golden section proportions fuse into one *Gestalt*. Strachan speaks of the overlapping of "visual and sonic affordances" within the DAW, and that visual affordances "serve to reconstruct thought about the conceptual ordering of music within the creative process" (Strachan 2017: 92). Writing before the inception of the DAW, Bonds articulates this visual overlap perfectly, "We know that musical form is in one sense temporal (…) the power of the synoptic image is so great that in our minds the form of a work can become a kind of imagined space in which the music operates" (Bonds 2010: 302). Therefore, does a DAW enhance that creative permeability between the visual and the aural – are the musically pleasing proportions of a song, for example, perceived in the writer's mind not only from their aural engagement with the unfolding work, but also visually?

Finally, the centrality of measuring time in the UI of the DAW may enhance the above permeability between the visual and the aural in the process of musical creativity. Through what Reuter describes as the advent of "new temporalities in the DAW" (Reuter 2012: 5), where digital production processes fundamentally impact our perception of, and engagement with musical time, the design of the DAW interface enables a dynamic and very real intersection of time with compositional musical activity: time – as clock time – is foregrounded in terms of the DAW's visual-spatial UI. For example, time is represented horizontally left to right, quantified numerically into minutes and seconds (or in the SMPTE time code graphic into hundredths of seconds), divided into grid structures, and timelines and tempo displays. This foregrounding may then directly shape how musicians perceive and manipulate time in the act of composing, and further impact upon matters of structural proportionality. Together with matters of composer intentionality previously mentioned, this issue might only be fully determined through undertaking observational case studies with DAW-based composers in the role of co-researchers (see for example, Burnard 2016; Collins 2001; Collins and Dunn 2011; Persson and Robson 1995).

In conclusion, in this study I have presented six varied pieces by Jacob Collier from across four albums which – through the use of a non-score based, clock time-contingent methodology – demonstrate a fascinating temporal correspondence with golden section proportionality. With the GS based as it is upon irrational numbers,

this proportionality appears to be defined by points of musical emphasis – those key event points which can include silences, sudden key shifts, new instrumental or vocal entries, climactic points and other expressive features – rather than correlating with a particular song structure, such as AABA, predicated on integer multiples of rational numbers such as four, eight and sixteen. This tentative opening up the study of GS proportionality in terms of temporal parameters, to encompass the world of popular music, reflects my overall aim, to "construct something that works cognitively, that fits together and handles new cases, that may implement further inquiry and invention" (Goodman 1978: 163). In this aim it has echoed Moore's concerns with "the wholesale importation of analytical methods borrowed from musical analysis and applied to popular music" (Moore 2003: 9) and postulated an alternative to simply cutting and pasting those notation-centric methodologies which have been derived from classical, predominantly European music tradition, onto contemporary musicological practice. Hopefully, such a move may confirm that this temporal understanding of the Golden Section in popular music offers "a rich set of possibilities for future research" (Žuvela 2011: 280).

Endnotes

(1) Alongside this range of scholarly publications, there are currently also a range of popular GS apps for example, Altrise - a "design tool for artists, designers, programmers, photographers. It allows you to design using the golden section property"; PhiMatrix – "apply the Golden Ratio in any kind of design"; Iratio- "the app is designed for graphic designers, architects, website designers (…)". A host of online videos explore the GS in car design.

(2) I am applying here what Lendvai refers to as the "positive" (long musical section followed by shorter section for example, 0.618/0.312) rather than the "negative" short musical section followed by longer section at 0.312/0.618 (Lendvai 1979: 20–21).

(3) The arrange window is one of several modes of the graphical representation of sound in a DAW where audio and MIDI parts are recorded, imported and organised as "regions" within a track listing for subsequent editing.

Acknowledgements

The author dedicates this article to the memory of Michael Dunn, without whom it would have never been realised.

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